FUNDAMENTALS OF PLASMA PHYSICS

Fundamentals of Plasma Physics is a rigorous explanation of plasmas relevant to controlled fusion, astrophysical plasmas, solar physics, and magnetospheric plasmas, plasma thrusters, and many other plasma applications. More thorough than previous texts, it exploits new, powerful mathematical techniques to develop deeper insights into plasma behavior.

The initial chapters develop the basic plasma equations from first principles and explore single particle motion with particular attention to adiabatic invariance, a concept that later recurs in many contexts. The author then examines the many types of plasma waves and the philosophically intriguing issue of Landau damping. Magnetohydrodynamic equilibrium and stability are then tackled with emphasis on the topological concepts of magnetic helicity and self-organization. More advanced topics follow, including magnetic reconnection, nonlinear waves, and the Fokker–Planck treatment of collisions. The book concludes by discussing non-neutral and dusty plasmas, and considers how these unconventional plasmas relate to conventional plasmas.

Written for beginning graduate students and advanced undergraduates, this text emphasizes the fundamental principles that apply across many different contexts. It is of interest to students and researchers in physics, astronomy, space physics, electrical engineering, and aeronautics.

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In 2001, Professor Bellan received the Popular Writing Award from the Solar Physics Division of the American Astronomical Society for an article he wrote for American Scientist. His previous book, *Spheromaks*, was published in 2000. He is a Fellow of the American Physical Society and has published over 80 papers in journals including *Physical Review Letters*, *Geophysical Research Letters*, *Physics of Plasmas*, *Nuclear Fusion*, and *The Review of Scientific Instruments*.

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To my parents

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Preface

This text is based on a course I have taught for many years to first-year graduate and senior-level undergraduate students at Caltech. One outcome of this experience has been the realization that although students typically decide to study plasma physics as a means towards some specific goal, they often conclude that the study of this subject has an attraction and charm of its own; in a sense the journey becomes as enjoyable as the destination. This conclusion is shared by me and I feel that a delightful aspect of plasma physics is the frequent transferability of ideas between extremely different applications so, for example, a concept developed in the context of astrophysics might suddenly become relevant to fusion or vice versa.

Applications of plasma physics are many and varied. Examples include controlled thermonuclear fusion, ionospheric physics, magnetospheric physics, solar physics, astrophysics, plasma propulsion, semiconductor processing, antimatter confinement, and metals processing. Furthermore, because plasma physics is extremely rich in both concepts and regimes, it has often served as an incubator for new ideas in applied mathematics. Concepts first developed in one of the areas listed above frequently migrate rather quickly to one or more of the other areas so it is very worthwhile to keep abreast of developments in areas of plasma physics outside of one's immediate field of interest. Dialog between plasma researchers in seemingly disconnected areas has often proved quite profitable for all concerned and it is my hope that this text will help to promote this interdisciplinary aspect.

The prerequisites for this text are a reasonable familiarity with Maxwell's equations, classical mechanics, vector algebra, vector calculus, differential equations, and complex variables – i.e., the contents of a typical undergraduate physics or engineering curriculum. Experience has shown that because of the many different applications for plasma physics, students studying plasma physics have a diversity of preparation and not all are proficient in all prerequisites. Brief derivations of many basic concepts are included to accommodate this range of preparation; these derivations are intended to assist those students who may have had little or no exposure to the concept in question and to refresh the memories of those who have had ample exposure but have forgotten the details. For example, rather than just invoke Hamilton–Lagrange methods or Laplace transforms, there is a quick derivation and then a considerable discussion showing how these concepts relate to plasma physics issues. These additional explanations make the book more self-contained and also provide a close contact with first principles.

The order of presentation and level of rigor have been chosen to establish a firm foundation and yet avoid unnecessary mathematical formalism or abstraction. In particular, the various fluid equations are derived from first principles rather than simply invoked and the consequences of the Hamiltonian nature of particle motion are emphasized early on and shown to lead to the powerful concepts of symmetry-induced constraint and adiabatic invariance. Symmetry turns out to be an essential feature of magnetohydrodynamic plasma confinement and adiabatic invariance turns out to be not only essential for understanding many types of particle motion, but also vital to many aspects of wave behavior.

The mathematical derivations have been presented with intermediate steps shown in as much detail as is reasonably possible. This occasionally leads to daunting-looking expressions, but it is my belief that it is preferable to see all the details rather than have them glossed over and then justified by an "it can be shown" statement.

The book is organized as follows: Chapters 1 to 3 lay out the foundation of the subject. Chapter 1 provides a brief introduction and overview of applications, discusses the logical framework of plasma physics, and begins the presentation by discussing Debye shielding and then showing that plasmas are quasi-neutral and nearly collisionless. Chapter 2 introduces phase-space concepts and derives the Vlasov equation and then, by taking moments of the Vlasov equation, derives the two-fluid and magnetohydrodynamic systems of equations. Chapter 2 also introduces the dichotomy between adiabatic and isothermal behavior, which is a fundamental and recurrent theme in plasma physics. Chapter 3 considers plasmas from the point of view of the behavior of a single particle and develops both exact and approximate descriptions for particle motion. In particular, Chapter 3 includes a detailed discussion of the concept of adiabatic invariance with the aim of demonstrating that this important concept is a fundamental property of all nearly periodic Hamiltonian systems and so does not have to be explained anew each time it is encountered in a different context. Chapter 3 also includes a discussion of particle motion in fixed frequency oscillatory fields; this discussion provides a foundation for later analysis of cold plasma waves and wave–particle energy transfer in warm plasma waves.

Chapters 4 to 8 discuss plasma waves; these are not only important in many practical situations, but also provide an excellent way for developing insight and intuition regarding plasma dynamics. Chapter 4 shows how linear wave dispersion relations can be deduced from systems of partial differential equations characterizing a physical system and then presents derivations for the elementary plasma waves, namely Langmuir waves, electromagnetic plasma waves, ion acoustic waves, and Alfvén waves. The beginning of Chapter 5 shows that when a plasma contains groups of particles streaming at different velocities, free energy exists that can drive an instability; the remainder of Chapter 5 then presents Landau damping and instability theory, which reveals that surprisingly strong interactions between waves and particles can lead to either wave damping or wave instability depending on the shape of the velocity distribution of the particles. Chapter 6 describes cold plasma waves in a background magnetic field and discusses the Clemmow–Mullaly–Allis diagram, an elegant categorization scheme for the large number of qualitatively different types of cold plasma waves that exist in a magnetized plasma. Chapter 7 discusses certain additional subtle and practical aspects of wave propagation including propagation in an inhomogeneous plasma and how the energy content of a wave is related to its dispersion relation. Chapter 8 begins by showing that the combination of warm plasma effects and a background magnetic field leads to the existence of the Bernstein wave, an altogether different kind of wave that has an infinite number of branches, and shows how a cold plasma wave can "mode convert" into a Bernstein wave in an inhomogeneous plasma. Chapter 8 concludes with a discussion of drift waves; these are ubiquitous low-frequency modes that have important deleterious consequences for magnetic confinement.

Chapters 9 to 12 describe plasmas from the magnetohydrodynamic point of view. Chapter 9 begins by presenting several basic magnetohydrodynamic concepts (vacuum and force-free fields, magnetic pressure and tension, frozen-in flux, and energy minimization) and then uses these concepts to develop an intuitive understanding for dynamic behavior. Chapter 9 then discusses magnetohydrodynamic equilibria and derives the Grad–Shafranov equation, an equation that depends on the existence of symmetry and characterizes three-dimensional magnetohydrodynamic equilibria. Chapter 9 ends with a discussion on accelerated magnetohydrodynamic flows such as occur in arcs, magnetoplasmadynamic thrusters, and astrophysical jets. Chapter 10 examines the stability of perfectly conducting (i.e., ideal) magnetohydrodynamic equilibria, derives the "energy principle" method for analyzing stability, discusses sausage and kink instabilities, and introduces the concepts of magnetic helicity and force-free equilibria. Chapter 11 examines magnetic helicity from a topological point of view and shows how helicity conservation and energy minimization lead to the Woltjer–Taylor model for magnetohydrodynamic self-organization. Chapter 12 departs from the ideal models presented earlier to discuss magnetic reconnection, a non-ideal behavior that permits the magnetohydrodynamic plasma to alter its topology and relax to a minimum-energy state.

Chapters 13 to 17 consist of various advanced topics. Chapter 13 considers collisions from a Fokker–Planck point of view and is essentially a revisiting of the issues in Chapter 1 using a more sophisticated analysis; the Fokker–Planck model is used to derive a more accurate model for plasma electrical resistivity and also to show the failure of Ohm's law when the electric field exceeds a critical value called the Dreicer limit. Chapter 14 considers two manifestations of wave–particle nonlinearity: (i) quasi-linear velocity-space diffusion due to weak turbulence and (ii) echoes. Quasi-linear diffusion is an extension of Landau damping to the nonlinear regime and shows how wave turbulence interacts with equilibria, while echoes validate in a dramatic fashion basic concepts underlying Landau damping and also instigate some interesting thoughts about the meaning of entropy. Chapter 15 discusses how nonlinear interactions enable energy and momentum to be transferred between waves, categorizes the large number of such wave–wave nonlinear interactions, and shows how these various interactions are all based on a few fundamental nonlinear coupling mechanisms. Chapter 16 discusses one-component plasmas (pure electron or pure ion plasmas) and shows how these plasmas have behaviors differing from conventional two-component, electron–ion plasmas. Chapter 17 discusses dusty plasmas, which are three-component plasmas (electrons, ions, and dust grains), and shows how the addition of a third component also introduces new behaviors, including the possibility of the dusty plasma condensing into a crystal. The analysis of condensation involves revisiting the Debye shielding concept and so corresponds in a sense to having the book end on the same note it started on.

Three appendices have been included: Appendix A describes an intuitive method for deriving the standard vector calculus identities, Appendix B uses vector calculus identities to provide a quick derivation of vector calculus operators in curvilinear coordinates, and Appendix C provides both a short list of physical constants and a summary of frequently used formulae with page references to the locations where these formulae were discussed.

I would like to extend my grateful appreciation to Professor Michael Brown at Swarthmore College for providing helpful feedback obtained from his using a draft version in a seminar course at Swarthmore and to Professor Roy Gould at Caltech for providing helpful insight into both the dynamics of non-neutral plasmas and the energetics of Debye shielding. I would also like to thank graduate students Deepak Kumar and Gunsu Yun for their careful scrutiny of the final drafts of the manuscript and for pointing out both ambiguities in presentation and

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