Remarkable Mathematicians

From Euler to von Neumann

Ioan James

Mathematical Institute, Oxford

THE MATHEMATICAL ASSOCIATION OF AMERICA

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MATHEMATICAL ASSOCIATION OF AMERICA
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Our first six remarkable mathematicians were born in the forty-six years from 1707 to 1752. Four came from France, one from each of Italy and Switzerland.

Leonhard Euler (1707–1783)

In mathematics the eighteenth century was the age of Euler, but before we come to him it is necessary to say a few words about that remarkable family, the Bernoullis. Originally from Antwerp, they left the Netherlands in the late sixteenth century to escape the Spanish persecution of Protestants and settled in Basel, where they married into the merchant class. Generation after generation they produced remarkable mathematicians, beginning with the brothers Jakob and Johann, who were born too early to qualify for

inclusion here. Johann's quarrelsome son Daniel falls just within our period, but like Isaac Newton he was even more remarkable as a physicist than as a mathematician. In mathematics he was overshadowed by his uncle Jakob and eclipsed by his friend Euler.

Leonhard Euler was born in Basel on April 15, 1707. His forbears were artisans, for the most part, but his father Paul Euler was a minister of the Protestant Evangelical Reformed Church. Paul Euler knew the Bernoullis well, since he and Johann had lived at Jakob's house when they were studying mathematics under him. Paul Euler's wife, Margarete Brucker, was also the daughter of a minister. The year after their son was born the family moved to the nearby village of Riehen, where his father was pastor. The boy Leonhard grew up with two younger sisters, Anna Maria and Maria Magdalena. After some early education at home he was sent back to the city to live with his maternal grandmother and attend the old-fashioned Latin school of Basel, where no mathematics was taught. In 1720, at the age of thirteen, he matriculated into the faculty of philosophy at the university. At that time the education offered was only mediocre. Euler mastered all the available subjects and graduated in 1722, at the age of fifteen. In the same year he competed, without success, for professorships in logic and in law.

The next year Euler transferred to the faculty of theology, in accordance with his father's wishes, but in addition to divinity he began to study mathematics seriously. As he wrote in his autobiographical sketch: 'I soon found an opportunity to be introduced to a famous professor Johann Bernoulli. True he was very busy and so refused flatly to give me private lessons; but he gave me much more valuable advice to start reading more difficult mathematical books on my own and to study them as diligently as I could; if I came across some obstacle or difficulty, I was given permission to visit him freely every Saturday afternoon and he kindly explained to me everything I could not understand.' Although Johann Bernoulli was recognized as one of the leading mathematicians in Europe his ordinary teaching was at an elementary level.

Euler received his master's degree in 1724 at the age of seventeen after writing a thesis comparing the natural philosophy of Descartes with that of Newton. By this time he had got to know several of the younger members of the Bernoulli clan, including Johann's son Daniel, seven years his senior. Johann himself became increasingly aware that his student was a genius. When he wrote to Euler in later years the salutations show his growing respect: in 1728 he addressed him as 'The very learned and ingenious young man'. The next year this had become 'The highly renowned and learned man', and the year after 'The highly renowned and by far most sagacious mathematician' until by 1745 he was addressing him as 'The incomparable L. Euler, the prince among mathematicians'.

Although Euler was a devout Calvinist he succeeded, with the support of Johann Bernoulli, in persuading his father that his true vocation was in mathematics, not the church. During the next two years, while seeking employment, Euler wrote his first important memoir, on the theory of acoustics. Also he entered a prize competition sponsored by the Académie Royale des Sciences in Paris (hereafter usually called the Paris Academy) concerning the masting of sailing ships, and received an honorable mention.

Prize competitions were an important feature of scientific life at least until the end of the nineteenth century. Originally they were a way of seeking solutions to specific problems. They usually emanated from the royal academies, notably those in Berlin and Paris, and although they provided an opportunity for an unknown young researcher it was quite normal for the well established to enter. In the case of the Paris Academy, for example, prizes were awarded for memoirs addressing specific problems in the mathematical or physical sciences. Among the rules of procedure, each entry had to be under a pseudonym or motto, accompanied by a sealed envelope similarly inscribed containing the name of the author, although this could often be guessed by the judges. The Bernoullis, particularly Daniel, were often successful in these competitions.

With the support of Johann Bernoulli, Euler applied for the vacant professorship of physics at the University of Basel, but was turned down, partly as being too young. The prospects of a position in Switzerland did not seem hopeful. However Daniel Bernoulli had recently accepted the offer of a senior appointment at the newly established Imperial Russian Academy of Sciences in St Petersburg (hereafter usually called the St Petersburg Academy). In 1725 he moved there with his elder brother Nicholaus, and two years later arranged for his young compatriot to join him. Initially Euler was given a junior post in the medical section of the academy, which meant that he had to spend a few months studying anatomy and physiology, but before long he managed to get transferred to the mathematical section, and became a member of the permanent staff. Euler lodged with Daniel Bernoulli and often worked with him at a time when his research interests lay mainly in mechanics and physics, particularly hydrodynamics.

Euler learnt the Russian language, and soon settled in to enjoy the social life of the great city. Unfortunately his arrival coincided with the death of Catherine the First, who had tried to continue the progressive policies of her late husband, the formidable Tsar Peter the Great; this was followed by a period of reaction and intolerance. When Daniel Bernoulli returned to Basel after six years in the Russian capital, Euler succeeded him as the premier mathematician at the academy. In the same year, feeling financially secure, he married Catharina Gsell, the daughter of a Swiss artist then working in Russia. They lived in a comfortable house beside the River Neva.

During this first St Petersburg period, which lasted fourteen years, Euler wrote both elementary and advanced mathematical textbooks for use in Russian schools, and solved many practical problems put to him by the Russian government. As well as being professor of mathematics he was also in charge of the department of geography, where one of his duties was to prepare a map of the country. However he was mainly occupied with mathematical research. Perhaps his best-known work of this period is his formulation of and solution to the problem of the seven bridges of Königsberg, which dates from 1736. This marks the beginning of the branch of mathematics known today as graph theory; Euler uses Leibniz' term 'analysis situs' in this connection, perhaps having heard it from one of the Bernoullis. During this time he wrote a treatise *Mechanica*, in which he showed that mathematical analysis could be applied systematically to Newtonian dynamics. In fact during this period he wrote almost ninety works for publication, and made notes of various important ideas to be developed later. He entered for the annual prize offered by the Paris Academy and was the winner no less than twelve times, surpassing even Daniel Bernoulli's record. It was during this period that he became blind in the right eye; it seems probable that the cause was scrofula (glandular tuberculosis).

Unfortunately conditions in Russia again became oppressive after the death of the Regent Anna Leopoldovna, mother of the infant Tsar Ivan the Sixth, and so, in 1741, Euler accepted an invitation from the King of Prussia, Frederick the Great, to move to the Prussian Royal Academy of Sciences in Potsdam (hereafter usually called the Berlin Academy), which had just been reorganized after a period of decline. As he was still paid a pension by the St Petersburg Academy, for all the next twenty-five years when he was in Berlin, effectively he was working for both. At first the Queen could extract from Euler only monosyllables in response to her enquiries. She taxed him with timidity and reserve, despite the cordiality of his reception: 'Why, then, will you not talk to me?' she asked. 'Because, Madam, I have just come from a country where people are hanged if they talk.' The Eulers settled down first in a house in Behrenstrasse, which is still preserved, and then acquired an estate in Charlottenburg, just outside the city.

It was during this period that he completed his masterpiece, the memoir on the calculus of variations called, in the lingua franca, *Methodus inveniendi lineas curvas maxime minimive proprietate gaudentes*. Its publication in 1744 led to his election to the Royal Society of London and to its Paris Academy, among other honours. In 1750 he conjectured the famous formula relating the number of faces, edges, and vertices of a convex polyhedron, and attempted to prove it. In a more popular vein he published his celebrated *Lettres à une Princesse d'Allemagne* (Letters to a German Princess), consisting of lessons in science for the King's niece, the Princess of Anhalt-Dessau. These 234 letters, written in the period 1760–2, were among the most successful popularizations of science in the eighteenth century but they were much more: apart from their ostensible purpose they provide us with the most exhaustive and authoritative treatment of natural philosophy written by a leading scientist in the eighteenth century. They were translated from German into several languages. But these are just a few examples of his prodigious output.

Frederick the Great saw science as the servant of the state. Its importance for him lay in its ability to further technological progress. Above all he sought to strengthen his army and enrich his lands. Frederick himself had little knowledge of theoretical science or mathematics, so he gauged the importance of the achievements of the Academy's scientists by their practical and military applicability. Yet despite the king's limited conception of science a great deal of theoretical scientific research was carried out at the Academy, since the practical problems he assigned to its staff were not unduly laborious and so plenty of time remained for independent work. 'I can do just what I wish [in my research]', Euler told a friend. 'The King calls me his professor, and I think I am the happiest man in the world.' Unfortunately this situation was not to last.

Initially it was the great French scientist Pierre Maupertuis who presided over the Academy. Following his death in 1759, Euler took over the management but under the direct supervision of the King, who did not entirely trust him. In addition to various substantial administrative responsibilities, Euler was asked to undertake various practical tasks, including finance, ballistics, navigation, water supply, and so on. He was also expected to advise the King on such matters as the purchase of scientific instruments, the construction of watermills, the administration of lotteries, the improvement of canals, and even on the construction of a stone wall around the garden of the academy. Although mostly absent from Berlin during the Seven Years War the Prussian monarch took a close interest in the administration of the Academy, particularly in the making of professional appointments. Eventually this created an impasse in his relationship with Euler, who expected some degree of academic freedom. Increasingly he fell out of favour with the King, who tended to look down on him, referring to him as 'my cyclops' in allusion to his loss of the sight of one eye. So in 1766, at the age of fifty-nine, Euler left Berlin, much to the King's displeasure, and returned to St Petersburg.

Throughout this second period in St Petersburg, which lasted until his death seventeen years later, the Empress Catherine the Great, herself of German origin, was on the Russian throne. She provided for him generously, even lent him one of her cooks. 'I and all others who had the good fortune to be some time with the Russian Imperial Academy', he wrote later, 'cannot but acknowledge that we owe everything which we are and possess to the favourable conditions which we had there.' However Euler's return to the Russian capital was dogged by misfortune at first. His house was burnt down and he lost many of his possessions. In addition he became almost totally blind due to an unsuccessful operation to remove a cataract in his one good eye. Fortunately Euler was blessed with a prodigious memory. As a boy he had memorized the entire text of Virgil's Aeneid. His ability to perform complex calculations in his head was well known, and his memory undoubtedly helped him to cope with blindness in the latter part of his life, one of the most fruitful periods of his career.

Euler's first wife Catharina died in 1776. Of their thirteen children, only three sons and two daughters survived beyond their early years. Euler was especially fond of children, often writing mathematics with a child on his lap. In 1777 he was married again to Catharina's half-sister, Salome Abigail Gsell. On September 18, 1783 he suffered a stroke and died at the age of seventy-six. Earlier that day he had given a mathematics lesson to one of his grandchildren, carried out some calculations on the ascent of balloons, and held a discussion with his assistants concerning the orbit of the newly discovered planet Uranus.

Euler's energies seemed inexhaustible. In pure mathematics his major fields were calculus, differential equations, analytic and differential geometry of curves and surfaces, number theory, infinite series, and the calculus of variations. In applied mathematics he created analytical mechanics. He wrote eminently readable textbooks on mechanics, algebra, mathematical analysis, analytic geometry, differential geometry, and the calculus of variations that were standard works for a century or more. In mathematical physics he built on the work of Daniel Bernoulli. In hydrodynamics, for example, he discovered the fundamental differential equations for the motion of an ideal fluid; and he applied them to the flow of blood in the human body. In the theory of heat he followed Daniel Bernoulli in regarding heat as an oscillation of molecules. He was one of the few scientists of the eighteenth century to favour the wave as opposed to the particle theory of light. He studied the propagation of sound and he obtained many results on the refraction and dispersion of light

However Euler was also remarkable for the skill with which he applied mathematics to practical problems. For example, he investigated the bending of beams and calculated the safety load of columns. He calculated the perturbative effect of celestial bodies on the orbits of planets. He calculated the paths of projectiles in resisting media. His three volumes on optical instruments contributed to the design of telescopes and microscopes. His work on the design of ships aided navigation. He produced a theory of the tides. Nor were his interests confined to subjects closely related to mathematics; he wrote about chemistry, geography, cartography, and much else.

No other mathematician has published as much as Euler did. He wrote almost 900 papers, memoirs, books, and other works. Of these almost half date from the second St Petersburg period, when he was almost blind and everything had to be dictated to assistants. It is estimated that of all the pages published on mathematics, mathematical physics, astronomy, and the engineering sciences during the last three-quarters of the eighteenth century, one third were written by Euler. Some 560 titles were published, but much remained unpublished during in his lifetime. The *Opera Omnia* have been appearing at the rate of one large volume a year on average for the last seventy years (the original programme of publication is now almost complete). But apart from his voluminous published works Euler left a mass of correspondence, personal diaries, and other papers which will continue to occupy the attention of scholars for many years to come.

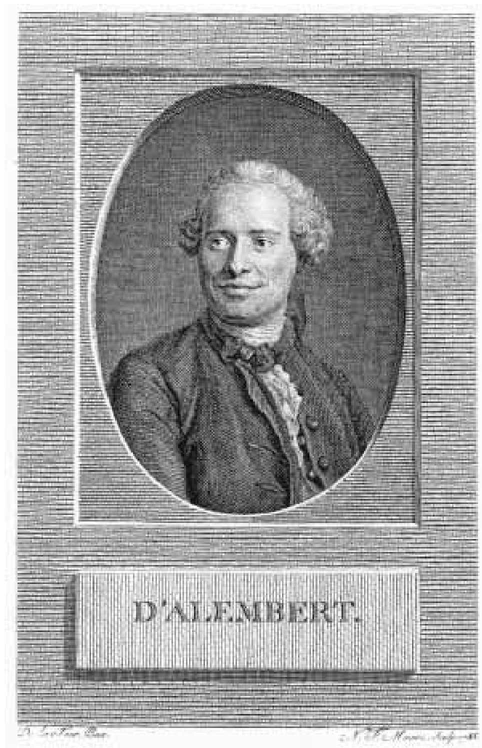
Euler was a simple man, not given to envy. As was said of Leibniz, he was glad to observe the flowering in other people's gardens of plants whose seeds he had provided. Euler had only a few immediate disciples, and none of them was a first-class scientist; yet, as said by Laplace, he was the teacher of all the mathematicians of his time. In mathematics the eighteenth century can fairly be labelled the age of Euler, but his influence on the development of mathematical sciences was not restricted to that period. The work of many outstanding nineteenth-century mathematicians arose directly from his.

Jean-le-Rond d'Alembert (1717–1783)

In the late seventeenth and early eighteenth centuries France had no mathematician to compare with the English Newton or the German Leibniz. However the relative mediocrity of French mathematics which marked the later part of the reign of Louis XIV was now followed by one of the brightest periods in all history, at a time when neither Britain nor Germany had any great mathematicians to show. D'Alembert, the first of the stars of French eighteenth-century mathematics, was often in dispute with Euler. Although Euler was the more powerful mathematician of the two he frequently exploited d'Alembert's ideas. However mathematics was only one of d'Alembert's many interests. He was a leading figure in the Enlightenment, the international movement which took on a special character in France.

Jean-le-Rond d'Alembert was born in Paris on November 17, 1717, so that he was ten years junior to Euler. He was the natural son of Claudine-Alexandrine Guérin, Marquise de Tencin, a well-known salon hostess and novelist of the period, and the Chevalier Louis-Camus Destouches-Canon, a cavalry officer. His mother, who was a lapsed nun, abandoned her new-born child on the steps of the church of Saint Jean-le-Rond, in the cloisters of the cathedral of Notre Dame, because she was afraid of being returned to her convent. However his father traced the child, who was being fostered in Picardy, and who had been given the Christian name of the church where he had been left. Destouches-Canon found him a home in Paris with an artisan named Rousseau and his wife. D'Alembert lived with them until he was forty-seven years old, and all his best scientific and literary work was done under their roof. Destouches-Canon also saw to the education of the boy, and when he died provided his son, who was then only nine years old, with a modest private income for the rest of his life. His mother, whose life was anything but virtuous, had nothing to do with him. The name d'Alembert may be a French version of the German Darenberg, but its significance is unknown.

D'Alembert attended the Collège de Quatre Nations (sometimes called after Mazarin, its founder), a Jansenist school offering a curriculum in the classics and rhetoric – and also offering more than the average amount of mathematics. In a break with tradition the mathematics lectures there were given in French, rather than Latin. The school possessed an excellent library,



of which the boy took full advantage. After obtaining the baccalaureate in 1735 he turned against a religious career, in spite of the efforts of his teachers to persuade him otherwise, and for a time studied both law and medicine before deciding on a career as a mathematician. Although he had received almost no formal scientific training, it is clear that on his own he had become familiar not only with Newton's works but also with those of l'Hôpital and other mathematicians of his day, particularly the Bernoullis.

In 1739, when he was twenty-two, d'Alembert sent his first communication to the Paris Academy. During the next two years he sent five more, dealing with methods of integrating differential equations and with the motion of bodies in resisting media. His communications were answered by Alexis Clairaut who, although only four years older than d'Alembert, was already an Academician. After several attempts to gain election, d'Alembert was successful in 1741.

For the next two years he worked on various problems in rational mechanics (what would nowadays be called theoretical mechanics) and published his *Traité de dynamique* (Treatise on dynamics). In this celebrated work he tried to formalize the new science of mechanics on Newtonian principles. In the first part of the treatise d'Alembert developed his own three laws of motion. Like others of his day he did not simply adopt those of Newton, which in any case were expressed in words, not symbols. He tried to show that the first two laws followed from basic ideas of space and time by reasoning which was mathematical rather than physical. It was not until he arrived at the third law that physical assumptions were involved; he implicitly assumed conservation of momentum, and eschewed the notion of force. The principle that bears his name can already be found in the work of Daniel Bernoulli. It is more like a convenient rule for using Newton's laws, and does not actually follow from them logically.

In 1744 d'Alembert published a companion volume, the *Traité de l'équilibre et du mouvement des fluides* (Treatise on the equilibrium and on the motion of fluids), treating the major problems of fluid mechanics then current. Clairaut published a competing work in the same year. D'Alembert's next work, *Réflexions sur la cause générale des vents* (Reflections on the general cause of the winds), although based on false assumptions, contained the first general use of partial differential equations in mathematical physics. This is just one of many instances where his methods were perfected by Euler. Another was his (incorrect) theory of vibrating strings, where we see the first appearance of a wave equation in physics. Then d'Alembert moved into celestial mechanics and this led him, in 1749, to publish his masterly *Recherches sur la précession des équinoxes et sur la nutation de la terre* (Investigations into the precession of the equinoxes and the nutation of the poles); nutation is a wobble of the earth's axis, due to lunar influences. During the next decade d'Alembert wrote one more major scientific work, his three-volume *Recherches sur différentes points importantes du système du monde* (Investigations into various important points concerning the solar system) of 1754/6. Devoted primarily to the motion of the moon it was written partially to guard his claims to originality against those of Clairaut. As often happened, d'Alembert's theory was the better one but Clairaut's methods were of more practical use to astronomers.

Having reached the top from humble beginnings d'Alembert did not want to lose his position. Once in the Academy, he had to struggle to stay ahead of his rivals. Whether by accident or inborn competitiveness, d'Alembert always seemed to be working on the same problems as other top mathematicians – initially Clairaut, later Daniel Bernoulli and Euler. He was always afraid of losing priority, and fell into a cycle of hasty publications followed by controversy over the meaning and significance of his work. Although Euler was the most able, this does not mean that the others did not make important contributions: because he was not much concerned with questions of priority on his own account, he was somewhat lax in acknowledging the contributions of others, and this caused a great deal of resentment. D'Alembert often had good cause to be annoyed, but he squandered his energy in fruitless arguments instead of giving his own theories the exposition they deserved. Many of his best ideas were not understood until Euler took them over.

D'Alembert submitted a memoir on fluid mechanics for a prize competition announced by the Berlin Academy and, although his entry was considered the best, the prize was not awarded. D'Alembert blamed this decision on Euler. Relations between the two men, already cool, now deteriorated further. The memoir in question was published in 1752 as 'Essay on a new theory of the resistance of fluids'. It was in this essay that d'Alembert expressed the differential equations of hydrodynamics in terms of a field and gave a description of the hydrodynamic paradox. Once again d'Alembert, Clairaut, Daniel Bernoulli, and Euler were all working on the same lines. Each of the four influenced the other three.

Despite the fact that he was an excellent writer on other subjects (the best of the age, according to Voltaire), his mathematics was seldom well presented. One of the reasons for d'Alembert's lack of attention to the exposition of his mathematical ideas was his involvement in the broader intellectual life of his time. In the 1740s he became one of the group of intellectuals known as the *philosophes*, joining in the mounting tide of criticism of the social and intellectual standards of the day. In these circles, mathematics was enjoying great prestige. Following Newton's success in explaining the motion of the planets it was hoped that rational enquiry, on the mathematical model, would allow the proper organization of all knowledge and the proper conduct of all human affairs. The movement to reorganize knowledge and conduct along rational lines, known as the Enlightenment, was particularly strong in France, where it was seen as a means to attack superstition in religion, reform the law, and later overthrow existing institutions. D'Alembert spent his time, much as the other *philosophes* did, working in the morning and afternoon of each day, and passing the evening attending the salons. Of slight build, with an expressive face, a high-pitched voice, a gift for conversation, and a talent for mimicry, he was known for his gaiety and wit; 'it is time I was weaned', he explained when he finally left his foster-home at the age of forty-seven.

The Enlightenment was not all talk, however, and one of its most solid achievements was the seventeen-volume *Encyclopaedia*, edited by Denis Diderot, which appeared between 1745 and 1772. D'Alembert wrote the important introduction to this huge work and in it summed up his views on the unity of all knowledge. It contributed greatly to the success of the *Encyclopaedia*, and was the main reason for his election to the Académie Française, the literary counterpart of the Académie des Sciences, in 1754. Eighteen years later he was elected secrétaire perpétuelle (permanent secretary) of this August body; one of his duties was to write the eulogy when any of the Academicians died. In spite of his efforts, the Académie Française failed to produce anything noteworthy in the way of literature during his period of office.

D'Alembert was also scientific editor of the *Encyclopaedia* for a time and wrote many of the mathematical articles. He divided the subject up into three main branches: pure, mixed, and physico-mathematics. Pure mathematics included arithmetic and geometry; mixed mathematics embraced mechanics, geometry, optics, acoustics, pneumatics, and the art of conjecturing or games of chance; physico-mathematics is where mathematical calculation is applied to experiment and seeks to deduce physical inferences whose certitude is close to geometric truth.

Eventually a rift developed among the Encyclopaedists between the materialists, led by Diderot, and the more moderate faction, led by Voltaire. Diderot leaned towards biology, for which he conjectured an absurd pseudo-mathematical basis, while deploring the 'impracticability' of ordinary mathematics. D'Alembert sided with Voltaire and severed his ties with the *Encyclopaedia* in 1758. In the 1760s intellectual fashion was moving away from mathematics, and d'Alembert found himself with only one other *philosophe* still interested in it, the probability theorist Condorcet. During the twenty years leading up to 1780, d'Alembert published a series of what he called *Opuscules mathématiques* (Mathematical essays), essays on all kinds of topics, largely going over what he had written before but with improvements.

D'Alembert seldom travelled, leaving France only once for a short visit in 1764 to the court of Frederick the Great. The Prussian monarch was enamoured of most things French. He had learned French as a child and never did master the German language. He saw the language of Louis XIV as the language of culture, and regarded German as crude. Thus it was to his friend Voltaire that he had turned for plans to build up the Berlin Academy shortly after his accession to the Prussian throne. On the recommendation of Voltaire the King tried to persuade d'Alembert to head it, but the latter declined the honour and recommended Euler instead. This helped to heal the rift that had grown up between the two.

Apart from scientific subjects d'Alembert also wrote about law and on religion. He went to visit Voltaire and, under the latter's influence, published anonymously a polemical book in which he called for the suppression of both the Jansenists and the Jesuits. He clashed with the social philosopher Jean-Jacques Rousseau. He wrote about music, especially on the composer Rameau's ideas concerning musical structure. While these activities have little to do with science, they serve to remind us of the wide spread of his interests.

The love of his life was Julie de Lespinasse, the cousin of Madame du Deffand, whose salon d'Alembert frequented. After a quarrel over poaching the salon's members, Julie set up a salon of her own, with his help. When Julie became ill with smallpox, d'Alembert nursed her back to health. When he himself fell sick, in 1765, she persuaded him to leave his foster home and move in with her. For the next ten years his life revolved around Julie's salon, and her death in 1776 came as a cruel blow. Humiliation was added to sorrow when he discovered from her letters that she had been passionately involved with other men throughout their time together.

D'Alembert spent the last seven years of his life in a small, rather dismal, apartment in the Louvre, to which he was entitled as permanent secretary of the Académie Française. He found himself unable to work on mathematics, although it was the only thing that still interested him. Lonely and bitter, he became gloomy about the future of mathematics itself. Nevertheless, he did what he could to support and encourage young mathematicians. The most notable achievement of his later years was to help launch the careers of Lagrange and Laplace, whose work in mechanics ultimately competed with much of his own. It must have given him some satisfaction to anticipate the future successes of his gifted protégés, even though they effectively ended the theory of mechanics as he knew it. What he could not have anticipated was that a minor element of his work, the use of complex numbers, would blossom in the next century, and enable mathematics to break out of the bounds set by eighteenth-century thought.

Jean-le-Rond d'Alembert died on October 29, 1783, at the age of sixty-five, just a few weeks after Euler. Perhaps he lived too long. Many of the other *philosophes* predeceased him, and those who remained alive in the 1780s were no longer the vibrant young revolutionaries they once had been.

What political success they achieved did not lead to tangible results. But to a large degree they had, in Diderot's phrase, changed the general way of thinking, and perhaps had some beneficial influence on the intellectual life of France. His collected literary works have been published, not his mathematical works.

D'Alembert was a mathematician in the tradition of Descartes, not a scientist like Euler. As he once said, mathematics owes its certainty to the simplicity of the things with which it deals, and this was tied up with his view of nature: however this was not just the view of a pure mathematician but included a wide range of applications. He was interested in questions of probability, life expectancy, and so on. He was ahead of his time in thinking about limits of functions and convergence of series, and almost alone in his day in regarding the differential as the limit of a function, the key concept around which calculus was eventually rationalized. Unfortunately his concept did not seem any more clear to his contemporaries than other schemes for explaining the nature of the differential. Finally, he seems to have been the first person on record to think of regarding time as a fourth dimension, since in 1754 he wrote: 'I said earlier that it is impossible to conceive of more than three dimensions. A clever acquaintance of mine believes that one might nevertheless consider timespan as a fourth dimension, and that the product of time with volume could in certain manner be a product of four dimensions; this idea may be contested, but it has, it seems to me, some merit, if only because of its novelty.' The clever acquaintance is thought to have been d'Alembert himself.

Joseph-Louis Lagrange (1736–1813)

Joseph-Louis Lagrange is particularly known for his uncompromisingly formal approach to analysis and mechanics. He viewed all functions as power series and attempted to reduce all mechanics to the analysis of such functions, without the use of geometry. At his death Lagrange left examples to follow, new problems to solve, and techniques to develop in all branches of mathematics. Bonaparte described him as the 'lofty pyramid' of the mathematical sciences.

In the eighteenth century, Turin was the capital of Piedmont, and the seat of the Savoyard Kings of Sardinia. Giuseppe Francesco Ludovico Lagrangia was treasurer of the Government Offices in Turin. His wife, Teresa Grosso, was the daughter of a physician in the small town of Cambiano, not far from Turin, and a member of the wealthy Conti family.

Their son, Joseph-Louis, was born there on January 25, 1736, one of eleven children, most of whom did not reach maturity. Despite the wealthy family background they were not well off, since his father had lost most of his money through gambling. His son eventually came to be thankful that he was not offered the chance to become a wealthy idler, declaring that if he had inherited a fortune he would probably not have cast his lot with mathematics. Because his father's family came from Touraine he regarded himself as French rather than Italian. His works were written in French and he used the names of Lagrange or La Grange rather than Lagrangia, but he spoke French with a distinct Italian accent all his life, and his favourite poet was Ariosto, according to d'Alembert.

Lagrange was intended by his father to become a lawyer. As a boy he raised no particular objections to this plan, but gradually decided he would prefer to study the exact sciences instead. At school he read the works of Euclid and Archimedes, without finding them particularly interesting, but the turning point came when he accidentally came across an article by the British astronomer and mathematician Edmund Halley arguing the superiority of calculus over Greek mathematics. After that he began to study the subject with such success that by the time he was nineteen he had been appointed professor of mathematics at the Royal School of Artillery in Turin.

In 1755 Lagrange began working on some of the isoperimetric problems that Euler and others were trying to solve at that time. The following year he applied the calculus of variations to mechanics and showed that, in the form of the principle of least action, it offered a general procedure for solving dynamical problems. He communicated these results to Euler who was greatly impressed, and withheld from publication a paper of his own which covered some of the same ground but used inferior methods. This was at the time when Euler was director of the mathematics division of the Berlin Academy. After Lagrange was elected associate foreign member of the Academy, on Euler's nomination, he considered the possibility of moving to Berlin, but decided to stay on in Turin for the time being. With some of his best students he organized a society for scientific research which soon developed into the Royal Academy of Sciences of Turin.

In 1759 the Turin Academy published its first volume of memoirs, to which Lagrange contributed three important papers. One was on the calculus of variations, another on the application of differential calculus to the theory of probability, the third provided a mathematical description of the vibrating string which settled in favour of Euler a controversy between d'Alembert and Euler. It demonstrated that Lagrange had carefully studied the works of Newton and others which have a bearing on this problem. Lagrange also contributed to later volumes of the memoirs, and in one of his papers we find, for the first time, the notion of eigenvalue for linear transformations.

Throughout eighteenth-century Europe, the scientific academies encouraged research into celestial mechanics, often offering prizes for the solution of specific problems. The main reason was that this kind of science was particularly useful for navigation. In 1764 Lagrange entered a competition sponsored by the Paris Academy to determine the gravitational forces that caused the moon to present a relatively unchanging face to the earth, and received the Grand prize. Two years later he again won the Grand prize, this time for a partial solution to a more complicated gravitational problem involving the planet Jupiter, its four then-known satellites, and the sun.

In 1763 Lagrange visited Paris; until then he had hardly gone far outside his native city. He was received with honour; unfortunately he became seriously ill during his stay. D'Alembert, who had been taking an interest in his career, believed that Lagrange was insufficiently appreciated in Turin. He used his influence with the result that the King of Sardinia and his ministers made fine promises but, when nothing came of them, d'Alembert turned towards Berlin and before long Lagrange received an attractive offer from Frederick the Great, expressing the wish of the 'greatest king in Europe' to have 'the greatest mathematician in Europe' resident at his court. Euler, as we know, was going back to St Petersburg. He offered Lagrange the opportunity to join him there, but Lagrange declined, and in 1766 became director of mathematical physics at the Berlin Academy instead. Before leaving for the Prussian capital he made another visit to Paris but again fell ill after a banquet in his honour and left without regret.

The next year Lagrange married a cousin, Vittoria Conti. In reply to an enquiry from his friend d'Alembert he wrote: 'I don't know whether I calculated ill or well, or rather, I don't believe I calculated at all; for I might have done as Leibniz did, who, compelled to reflect, could never make up his mind. I confess to you that I never had a taste for marriage . . . but circumstances decided me to engage one of my young kinswomen to take care of me and all my affairs. If I neglected to inform you it was because the whole thing seemed to me so inconsequential in itself that it was not worth the trouble of informing you of it.' However the marital relationship deepened over the years, and when his wife Vittoria died sixteen years later after a lingering illness Lagrange was heartbroken.

For mathematical research the Berlin years were fruitful. Lagrange was not required to lecture, but he was composing memoirs nearly every month on subjects ranging from probability to the theory of equations. In 1767 he published an important memoir 'On the solution of numerical equations'; four years later presented another entitled 'Reflections on the algebraic resolution of equations' to the Berlin Academy, where the point of view anticipates that of Galois. In number theory Lagrange solved some of the problems posed by Fermat, including the famous theorem that every positive integer is the sum of the squares of four integers. He continued his investigation of gravitational interactions between planetary bodies, winning, in 1772, his third Grand prize from the Paris Academy for a memoir on attractions between the sun, moon, and earth. In 1774 and in 1778 he again won Grand prizes, bringing his total up to four, the first for work related to lunar movement, the second for a study of the perturbations of comets.

During his twenty years at the Berlin Academy, Lagrange worked on what he called *Méchanique Analitique* (Analytical Mechanics), the application of calculus to the motion of rigid bodies. His conclusions were organized into a volume under that title published in 1788. To quote from the preface:

> We already have various treatises on mechanics but the plan of this one is entirely new. I have set myself the problem of reducing this science [*mechanics*], and the art of solving the problems pertaining to it, to general formulae whose simple development gives all the equations necessary for the solutions of each problem . . . No diagrams will be found in this work. The methods which I expound in it demand neither constructions nor geometrical or mechanical reasonings, but solely algebraic *[analytic]* operations subjected to a uniform and regular procedure. Those who like analysis will be pleased to see mechanics become a new branch of it, and will be obliged to me for having extended its domain.

The *Méchanique Analitique* was his masterpiece, a scientific poem according to Hamilton. About 1774 there was talk of returning to Italy, possibly to Turin, or to Naples, where he might have become director of the newly established Academy, but nothing came of it. Around 1780 Lagrange developed depression and lost interest in mathematics generally for some years. 'I begin to feel the pull of my inertia increasing little by little, and I cannot say I shall still be doing mathematics ten years from now', he wrote to d'Alembert, 'it also seems to me that the mine is already too deep, and that unless new veins are discovered it will have to be abandoned.' D'Alembert, thirteen years his senior, wrote back, 'In God's name do not renounce work, for you the strongest of all distractions. Good-bye, perhaps for the last time. Keep some memory of the man who of all in the world cherishes and honours you the most.'

Following the death of Frederick the Great in 1786 an indifference towards science and a resentment of foreigners arose in Berlin; meanwhile the Italian princes redoubled their efforts to attract Lagrange to their courts. However it was an offer from Paris which finally persuaded him to leave Berlin, after twenty very successful years. In July 1787 he became a *pensionnaire vétéran* of the Paris Academy, of which he had been a foreign associate member since 1772. In the French capital he was received with every mark of distinction; apartments in the Louvre were set aside for his reception. He was always welcome at social and scientific gatherings. However his depression worsened; he was known to gaze out of the window for long periods of time. He told his friends and colleagues that mathematics was no longer important to him.

In 1792, at the age of fifty-six, Lagrange took as his second wife Renée-Françoise-Adelaide Le Monnier, the teenage daughter of a friend and colleague of his. His new bride was devoted to him, and it is said she helped Lagrange gradually regain his interest in mathematics and life generally. However the second marriage was childless like the first.

During the revolutionary period Lagrange managed to remain politically neutral, although he was granted a pension by the revolutionary government and served on the Commission charged with the establishment of standards for weights and measures, out of which the metric system emerged. We shall learn more about this commission in the profile of Laplace. In 1793 laws came into effect which meant the arrest of all foreigners born in enemy countries and the confiscation of their property; but Lagrange was specifically exempted. The academies were suppressed, as we shall learn in the next profile, but as new institutions succeeded them positions were always found for Lagrange, whatever the government in power. In 1795 he was appointed professor of mathematics at the new Ecole Normale, and after that closed he became professor at the Ecole Polytechnique. He wrote two textbooks on calculus for students, trying to organize and systematize what was still a difficult and incoherent set of methods.

Lagrange was known for his gentle demeanour and his diplomatic skills. In appearance he was of medium height, and of slight build, with pale blue eyes and a colourless complexion. In character he was nervous and timid, detested controversy and to avoid it allowed others to take the credit for what he himself had done. At the Berlin Academy, he remained in favour with the King, unlike Euler. When he first settled in Paris he was doted on by Queen Marie-Antoinette, yet later he managed to remain on good terms with leaders of the French Revolution and later still to find favour with Bonaparte, who consulted him frequently. He was appointed Senator, a count of the Empire, and Grand Officer of the Legion of Honour. In his seventies he began revising and extending his masterpiece, the *Méchanique Analitique*, for a second edition. However the long hours of work diminished his strength and energy and he suffered increasingly from fainting spells. He died on April 11, 1813, at the age of seventy-seven. His body was brought to rest in the Panthéon, in recognition of his contributions to science.

Lagrange kept himself well informed about the works of other mathematicians. His close friendship with d'Alembert, with whom he frequently exchanged letters, should not obscure the striking divergence of their ideas. D'Alembert's mathematical production was characterized by a realism which links him to Newton and Cauchy. Lagrange, on the other hand, displayed in his youth, and sometimes in his later years, a poetic sense that recalls the creative audacity of Leibniz.

Laplace, as we shall see later, was a complete contrast to Lagrange in many respects, and one has the distinct impression that Lagrange did not care for him. In reply to a rather pompous letter from Laplace about the importance of his own work, Lagrange wrote: 'I have always regarded mathematics as an object of amusement rather than of ambition, and I can assure you that I enjoy the works of others much more than my own, with which I am always dissatisfied. You will see by that, if you are exempt from jealousy by your own success, I am none the less so by my disposition.' The story is told of how one day, after Laplace had invited Lagrange to dinner, Lagrange asked, 'will it be necessary to wear the costume of a senator?', in a mocking tone, of which everyone except Laplace sensed the malice. Although Lagrange was always reserved towards Euler, whom he never met, it was Euler, among the older mathematicians, who influenced him most. That is why any study of his work must be preceded by or accompanied by an examination of the work of Euler. Yet even in the face of this great model he preserved an originality that allowed him to criticize, but above all to generalize, to systematize, and to deepen, the work of his predecessors.

Gaspard Monge (1746–1818)

The educational reforms that began during the French Revolution had a profound effect on the future development of science in France. Gaspard Monge played a leading role in this movement, especially in the creation of the Ecole Polytechnique, the illustrious foundation later to inspire the Swiss Federal Technische Hochschule, the Massachusetts Institute of Technology, and other great institutions. He also helped to devise the metric system, which has spread throughout the world. The descriptive geometry which he largely created remains of practical importance. In pure mathematics he may be regarded as the father of differential geometry.

Jacques Monge was a small trader operating in Beaune, the centre of the wine-producing Côte d'Or region of Burgundy. Although his wife Jeanne (née Rousseau) was a native Burgundian, he originally came from the Savoy. Their eldest son Gaspard was born on May 9, 1746. In spite of their modest circumstances the couple made exceptional efforts to ensure