

Graded exercises in  
Advanced level mathematics

# **Graded exercises in pure mathematics**

Edited by Barrie Hunt



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WJEC	Welsh Joint Education Committee
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## 0

## Background knowledge

## 0.1 Basic arithmetic – highest common factor; lowest common multiple; fractions

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \quad \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

1 Express as a product of prime factors:

- (a) 30      (b) 49      (c) 53      (d) 84  
 (e) 108      (f) 693      (g) 1144      (h) 14 553

2 Find the highest common factor (HCF) of:

- (a) 6, 10                      (b) 7, 14                      (c) 30, 42  
 (d) 24, 40, 64              (e) 42, 70, 182              (f) 169, 234, 299  
 (g) 252, 378, 567              (h) 51, 527, 1343

3 Find the lowest common multiple (LCM) of:

- (a) 6, 10      (b) 7, 14      (c) 30, 42      (d) 2, 3, 4  
 (e) 5, 25      (f) 5, 7, 11      (g) 4, 21, 22      (h) 14, 18, 21

4 Express each fraction in its lowest terms, without using a calculator:

- (a)  $\frac{7}{35}$       (b)  $\frac{15}{125}$       (c)  $\frac{26}{39}$       (d)  $\frac{16}{80}$   
 (e)  $\frac{81}{108}$       (f)  $\frac{3a}{12a}$       (g)  $\frac{42a^2}{56a}$       (h)  $\frac{22ab^2}{121b}$

5 Complete:

- (a)  $\frac{3}{4} = \frac{\quad}{24}$       (b)  $\frac{4}{5} = \frac{\quad}{20}$       (c)  $\frac{4}{7} = \frac{\quad}{21}$       (d)  $\frac{7}{8} = \frac{\quad}{64}$   
 (e)  $\frac{7}{4} = \frac{\quad}{20}$       (f)  $\frac{2a}{3} = \frac{\quad}{9}$       (g)  $\frac{a}{b} = \frac{\quad}{bx}$       (h)  $\frac{2}{a} = \frac{\quad}{a^2}$

6 Simplify, without using a calculator:

- (a)  $\frac{3}{4} + \frac{2}{3}$       (b)  $\frac{2}{7} - \frac{1}{5}$       (c)  $\frac{4}{13} + \frac{2}{7}$       (d)  $\frac{5}{12} - \frac{3}{8}$

$$(e) 1\frac{3}{4} + 2\frac{7}{8} \quad (f) 5\frac{2}{3} - 3\frac{1}{9} \quad (g) 2\frac{1}{7} + \frac{3}{4} \quad (h) 3\frac{2}{5} + 2\frac{2}{3}$$

7 Express as a single fraction:

$$(a) \frac{3a}{4} + \frac{2a}{3} \quad (b) \frac{2a}{7} - \frac{a}{5} \quad (c) \frac{3}{a} + \frac{2}{a} \quad (d) \frac{3}{a} + \frac{2}{b}$$

$$(e) \frac{1}{u} + \frac{1}{v} \quad (f) \frac{5}{a} - \frac{2}{a^2} \quad (g) p - \frac{2}{q} \quad (h) \frac{3}{ab} - \frac{5}{ac}$$

8 Without using a calculator, simplify and express each fraction in its lowest terms:

$$(a) 6 \times \frac{2}{3} \quad (b) \frac{1}{2} \times \frac{3}{4} \quad (c) \frac{3}{5} \times \frac{4}{7} \quad (d) \frac{3}{5} \times \frac{4}{9}$$

$$(e) \frac{3a}{7} \times \frac{2}{5a} \quad (f) \frac{4a^2}{11} \times \frac{3}{2ab} \quad (g) x \times \frac{1}{x} \quad (h) x^2 \left( \frac{3}{x} + \frac{2}{x^2} \right)$$

9 Without using a calculator, simplify and express each fraction in its lowest terms:

$$(a) 6 \div \frac{2}{3} \quad (b) \frac{1}{2} \div \frac{3}{4} \quad (c) \frac{3}{5} \div \frac{6}{25} \quad (d) \frac{3}{5} \div \frac{4}{9}$$

$$(e) \frac{3a}{7} \div \frac{2a}{5} \quad (f) \frac{4}{11a^2} \div \frac{2}{3ab} \quad (g) x \div \frac{1}{x} \quad (h) \frac{1}{x^2} \div \frac{1}{x}$$

10 Which is larger,  $\frac{77}{78}$  or  $\frac{78}{79}$ ?

11 (a) The fraction  $\frac{20}{91}$  is written as  $\frac{1}{7} + \frac{1}{a}$ . Find  $a$ .

(b) Calculate:

$$(i) \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)$$

$$(ii) \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \dots \left(1 - \frac{1}{n}\right)$$

12 Find the greatest number which, when divided into 1407 and 2140, leaves remainders of 15 and 23 respectively.

## 0.2 Laws of indices

$$a^m \times a^n = a^{m+n} \quad \frac{a^m}{a^n} = a^{m-n} \quad (a^m)^n = a^{mn}$$

1 Simplify:

$$(a) a^3 \times a^4 \quad (b) a^7 \times a^6 \quad (c) a \times a^3$$

$$(d) 2a^3 \times 3a^2 \quad (e) 5a^2 \times a^7 \quad (f) \frac{2}{3}a^3 \times 6a^4$$

2 Simplify:

$$(a) \frac{x^9}{x^2} \quad (b) \frac{p^4}{p^3} \quad (c) \frac{x^{12}}{x} \quad (d) \frac{12a^7}{4a^2} \quad (e) \frac{12a^5}{8a^3} \quad (f) \frac{2a^2b}{6ab^2}$$



3 Simplify:

(a)  $(a^5)^3$       (b)  $(2a)^4$       (c)  $(5a^3)^2$       (d)  $5(a^3)^2$   
 (e)  $(-2a^2)^4$       (f)  $(3a^2b^3)^3$

4 Simplify:

(a)  $\sqrt{x^2}$       (b)  $\sqrt{x^6}$       (c)  $\sqrt{a^2b^2}$       (d)  $\sqrt{4a^2}$   
 (e)  $\sqrt[3]{-x^6}$       (f)  $\sqrt{9a^{10}b^4}$

5 Expand:

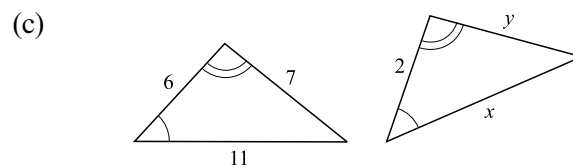
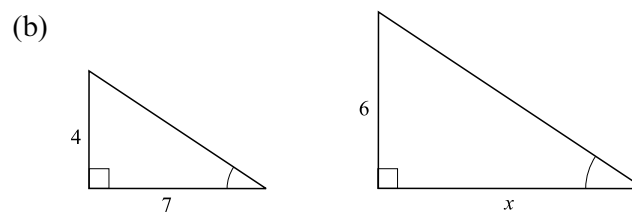
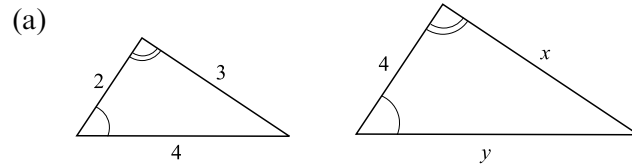
(a)  $(1 + x^2)^2$       (b)  $(3 - a^3)^2$       (c)  $\left(x^2 - \frac{1}{x^2}\right)^2$

6 Simplify:

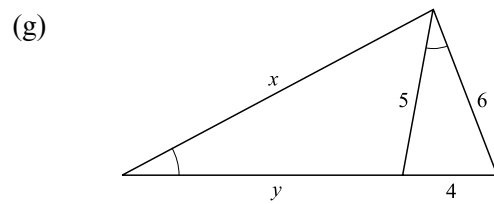
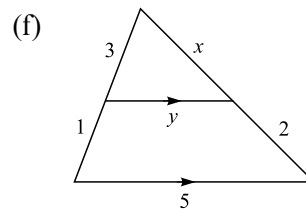
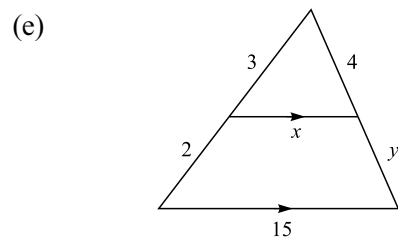
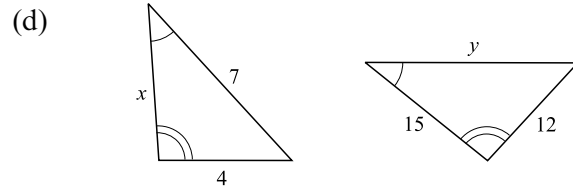
(a)  $\frac{x^2 + x^5}{x}$       (b)  $\frac{3x^8 + 2x^4}{x^4}$   
 (c)  $3x^2 + (5x)^2 - \frac{3x^3}{x}$       (d)  $\frac{10x^2y + 6xy^2 - 8x^2y^2}{2xy}$

### 0.3 Similar figures

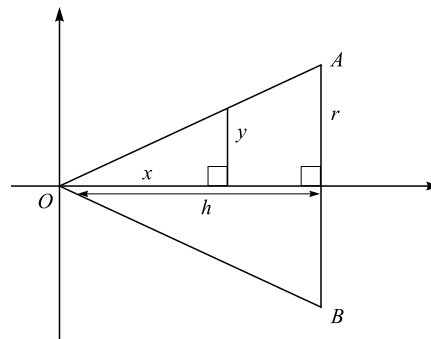
1 Find the sides marked  $x$  and/or  $y$  in each of the following pairs of similar triangles.



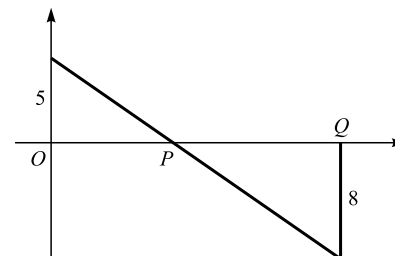
6 Background knowledge



2  $OAB$  is the cross-section of a cone, radius  $r$ , height  $h$ .  
Express  $y$  in terms of  $r$ ,  $h$  and  $x$ .



3 The coordinates of  $Q$  are  $(4, 0)$ .  
What are the coordinates of  $P$ ?



4 A sphere has radius 8 cm and a second sphere has radius 12 cm.  
What is the ratio of their (a) areas, (b) volumes?

- 5 A solid metal cylinder of radius 6 cm and height 12 cm weighs 6 kg. A second cylinder is made from the same material and has radius 8 cm and height 16 cm. How much does this cylinder weigh?
- 6 A liquid is poured into a hollow cone, which is placed with its vertex down. When  $400 \text{ cm}^3$  has been poured in, the depth of water is 100 cm. What is the depth of water after (a)  $1000 \text{ cm}^3$ , (b)  $x \text{ cm}^3$  has been poured in? Plot the graph to show how depth varies with volume.

#### 0.4 Basic algebra – multiplying brackets, factorising quadratics, solution of simultaneous equations

$$(a + b)(c + d) = ac + ad + bc + bd$$

1 Expand:

- (a)  $3(4 + a)$       (b)  $6(2 - 3a)$       (c)  $a(a + 3)$   
 (d)  $a(2a + 3b)$       (e)  $3a(5a - 2b)$       (f)  $x\left(2 + \frac{3}{x}\right)$

2 Multiply out the brackets:

- (a)  $(x + 2)(x + 5)$       (b)  $(x - 3)(x + 4)$       (c)  $(2x + 1)(3x + 5)$   
 (d)  $(5x - 2)(5x + 2)$       (e)  $(3a + 2)^2$       (f)  $(p + 3q)(2p - 5q)$   
 (g)  $\left(x + \frac{2}{x}\right)^2$       (h)  $(2x^2 + 1)(x + 3)$

3 Factorise:

- (a)  $4x + 8y$       (b)  $x^2 - 3x$       (c)  $5x^2 + 2xy$   
 (d)  $2\pi r^2 + 2\pi rh$       (e)  $ut + \frac{1}{2}at^2$       (f)  $2x^3 + 3x^4$

4 Factorise:

- (a)  $x^2 + 4x + 3$       (b)  $x^2 + 2x - 3$       (c)  $a^2 - 6a + 9$   
 (d)  $x^2 + 7x + 10$       (e)  $p^2 + p - 30$       (f)  $2a^2 + 7a + 3$   
 (g)  $6y^2 - 7y - 5$       (h)  $p^2 - 4q^2$       (i)  $p^2 + 4pq - 12q^2$   
 (j)  $15p^2 - 34pq - 16q^2$       (k)  $9x^2 + 30xy + 25y^2$       (l)  $10a^2 + 31a - 14$

5 Simplify:

- (a)  $\frac{3x + 6}{3}$       (b)  $\frac{x^2 + 2x}{x}$       (c)  $\frac{x^2 + 3x + 2}{x + 1}$       (d)  $\frac{16 - x^2}{x + 4}$

6 Solve the simultaneous equations:

- (a)  $x + y = 4$       (b)  $x + 2y = 8$       (c)  $2x + 3y = 2$   
        $x - y = -6$              $x + 5y = 17$              $x - 2y = 8$

8 Background knowledge

- (d)  $3x - 2y = 1$   
 $-5x + 4y = 3$
- (e)  $2x + 5y = -14$   
 $3x + 2y = 1$
- (f)  $5x - 3y = 23$   
 $7x + 4y = -17$
- (g)  $4x - 3y = 0$   
 $6x + 15y = 13$
- (h)  $2x + 3y + 4 = 0$   
 $5x - y - 7 = 0$

7 Multiply out the brackets:

- (a)  $(x - 1)(x^2 + x + 1)$  (b)  $(a + b)^3$  (c)  $(a + b)^4$   
(d)  $(x + \sqrt{2})(x - \sqrt{2})$

8 Simplify:

- (a)  $(a + b)^2 - (a - b)^2$  (b)  $\frac{x^3 + 2x^2 + x}{x^2 + x}$  (c)  $\frac{x^4 - 13x^2 + 36}{(x - 2)(x^2 - 9)}$

9 Solve the pairs of simultaneous equations below, explaining your results graphically.

- (a)  $2x + 3y = 8$   
 $6x + 9y = 12$
- (b)  $2x + 3y = 8$   
 $6x + 9y = 24$

## 0.5 Solving equations; changing the subject of a formula

1 Solve the following equations.

- (a)  $2x + 1 = 7$  (b)  $2 - 3x = 8$   
(c)  $5x + 2 = 3x - 5$  (d)  $6x + 3 = 8 - 2x$   
(e)  $3(x + 2) = 9x$  (f)  $4(2x - 7) = 3(5x + 1)$   
(g)  $x^2 = 81$  (h)  $x^2 - 25 = 0$   
(i)  $x = \frac{16}{x}$  (j)  $x^3 + 27 = 0$   
(k)  $x^2 = 7x$  (l)  $x - \frac{4}{x} = 0$   
(m)  $x(x - 4) = 0$  (n)  $(x + 3)(x - 7) = 0$   
(o)  $(2x - 3)(x + 4)(3x + 2) = 0$

2 Rearrange to make the given variable the subject of the formula:

- (a)  $Q = CV$  ( $C$ ) (b)  $C = 2\pi r$  ( $r$ )  
(c)  $F = \frac{9}{5}C + 32$  ( $C$ ) (d)  $y = mx + c$  ( $m$ )  
(e)  $P = 2(\ell + w)$  ( $\ell$ ) (f)  $S = \frac{1}{2}n(a + \ell)$  ( $a$ )  
(g)  $v^2 = u^2 + 2as$  ( $a$ ) (h)  $s = ut + \frac{1}{2}at^2$  ( $a$ )  
(i)  $u = a + (n - 1)d$  ( $d$ ) (j)  $s = \frac{n}{2}\{2a + (n - 1)d\}$  ( $d$ )

**3** Rearrange to make the given variable the subject of the formula:

- (a)  $E = mc^2$  ( $c$ )                      (b)  $V = \frac{4}{3}\pi r^3$  ( $r$ )  
 (c)  $V = \frac{1}{3}\pi r^2 h$  ( $r$ )                      (d)  $y = \frac{4}{x^2}$  ( $x$ )  
 (e)  $I = \frac{1}{2}m(v^2 - u^2)$  ( $v$ )              (f)  $y = 2\sqrt{x} + 3$  ( $x$ )  
 (g)  $T = 2\pi\sqrt{\frac{\ell}{g}}$  ( $\ell$ )                      (h)  $A = \pi(r^2 - r_1^2)$  ( $r$ )  
 (i)  $y = \frac{1}{x-a}$  ( $x$ )                              (j)  $c = \sqrt{a^2 + b^2}$  ( $a$ )

**4** In each case, show clearly how the second formula may be obtained from the first.

- (a)  $I = \frac{iR}{R+r}, \quad r = \frac{(i-I)R}{I}$   
 (b)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad y = \frac{b}{a}\sqrt{(a^2 - x^2)}$   
 (c)  $y = \frac{x-2}{x}, \quad x = \frac{2}{1-y}$   
 (d)  $y = \frac{3x+2}{5-x}, \quad x = \frac{5y-2}{y+3}$   
 (e)  $I = \frac{Er}{R+r}, \quad r = \frac{IR}{E-I}$   
 (f)  $\frac{1}{R} = \frac{1}{u} + \frac{1}{v}, \quad v = \frac{Ru}{u-R}$

**5** The surface area,  $S$ , of a cylinder is given by  $S = 2\pi r^2 + 2\pi rh$ . Its volume,  $V$ , is given by  $V = \pi r^2 h$ . Express  $V$  in terms of  $S$  and  $r$  only.

## 0.6 The straight line $y = mx + c$ ; gradient and intercept

The line  $y = mx + c$  has gradient  $m$ , intercept  $c$

**1** Plot the graph of  $y = 4x + 2$  for  $-3 \leq x \leq 3$ . Calculate the gradient of the line.  
Write down where it crosses the  $y$ -axis (the  $y$ -intercept).

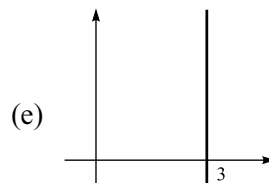
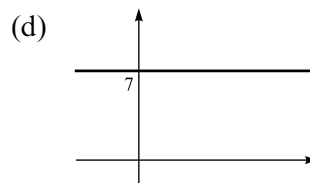
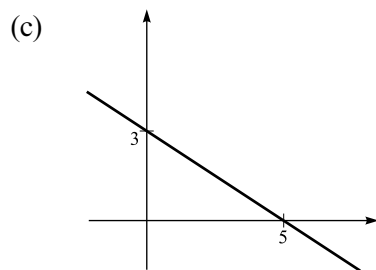
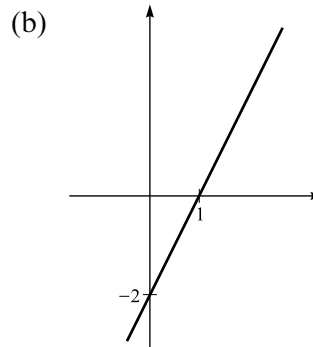
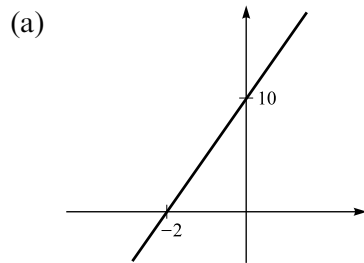
2 Complete the table.

	Equation	Gradient	Intercept
(a)	$y = 5x - 2$		
(b)	$y = 1 - 3x$		
(c)	$y = \frac{1}{2}x$		
(d)	$y = -4 - 3x$		
(e)		2	5
(f)		6	-2
(g)		7	$\frac{1}{2}$
(h)		1	0
(i)	$2y = 4x + 1$		
(j)	$5y = 2x$		

3 Sketch the following lines.

(a)  $y = 2x + 5$       (b)  $y = \frac{1}{2}x + 2$       (c)  $y = -x$       (d)  $y = -x + 1$

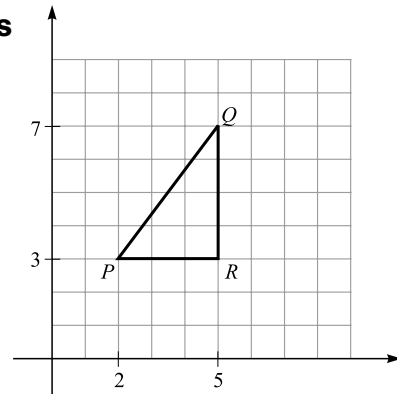
4 Write down the equation of each of the lines shown.



- 5 Find the equation of the line perpendicular to  $y = 2x - 1$  which passes through  $(0, 3)$ .
- 6 State the coordinates of the point where the line  $\frac{y}{4} + \frac{x}{6} = 1$  crosses
  - (a) the  $x$ -axis, (b) the  $y$ -axis.

**0.7 The distance between two points**

- 1 (a)  $P$  and  $Q$  are two points with coordinates  $(2, 3)$  and  $(5, 7)$  respectively. By applying Pythagoras' theorem to triangle  $PQR$ , find the distance  $PQ$ .  
 (b) By drawing a suitable diagram, find a formula for the distance  $PQ$  where  $P$  and  $Q$  have coordinates  $(x_1, y_1), (x_2, y_2)$  respectively.

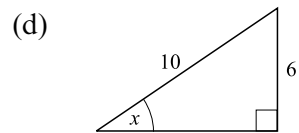
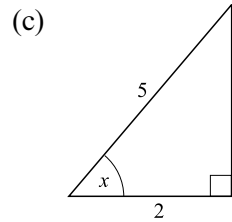
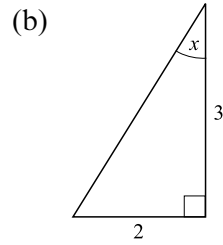
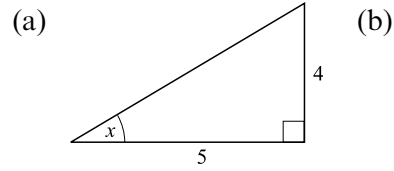


- 2 Find the distance between the following pairs of points.
  - (a)  $(1, 2), (6, 14)$       (b)  $(3, 2), (6, 3)$       (c)  $(-1, 4), (2, 7)$
  - (d)  $(4, 2), (1, -3)$
- 3 Show that the triangle with vertices at  $(1, 0), (3, 0), (2, \sqrt{3})$  is equilateral.
- 4 Which of the points  $(6, 4), (-3, 6), (2, -4)$  is nearest to  $(1, 2)$ ?
- 5 Find the distance of the point  $P(x, y)$  from (i)  $O(0, 0)$  (ii)  $R(4, 3)$ .  
 If  $P$  is equidistant from  $O$  and  $R$ , find the equation of the locus of  $P$ .

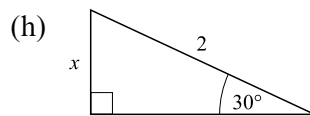
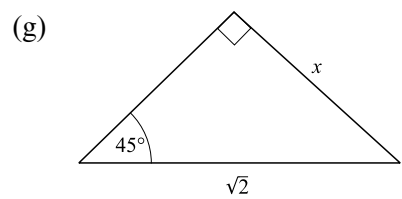
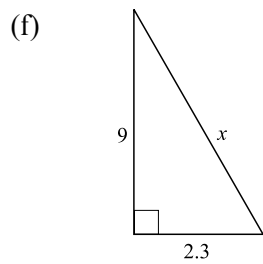
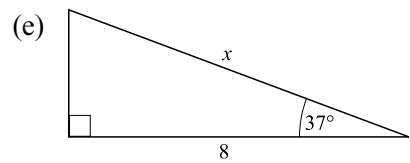
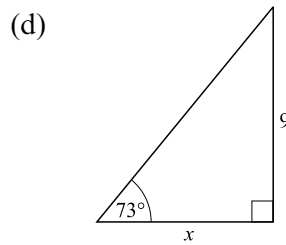
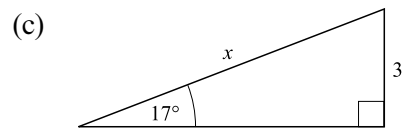
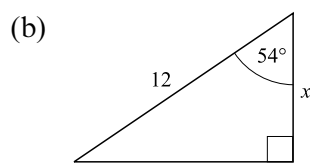
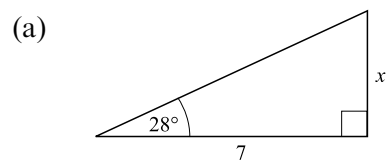
**0.8 Trigonometry – right-angled triangles; sine and cosine rules**

<p>In right-angled triangles:  <b>Pythagoras' theorem</b> <math>a^2 + b^2 = c^2</math>  <math>\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{a}{c}, \quad \cos A = \frac{\text{adj}}{\text{hyp}} = \frac{b}{c},</math>  <math>\tan A = \frac{\text{opp}}{\text{adj}} = \frac{a}{b}</math></p> <p>In all triangles:                  sine rule <math>\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}</math>                  cosine rule <math>a^2 = b^2 + c^2 - 2bc \cos A</math></p>	
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1 Find the angles marked  $x$ .

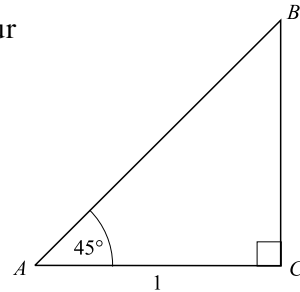


2 Find the sides marked  $x$ .

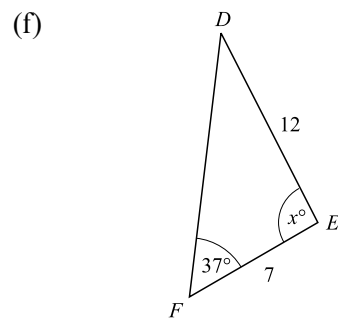
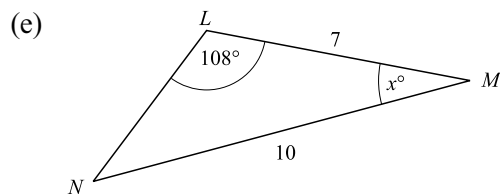
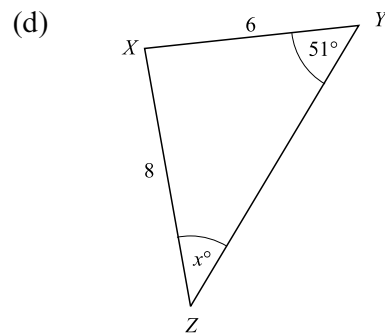
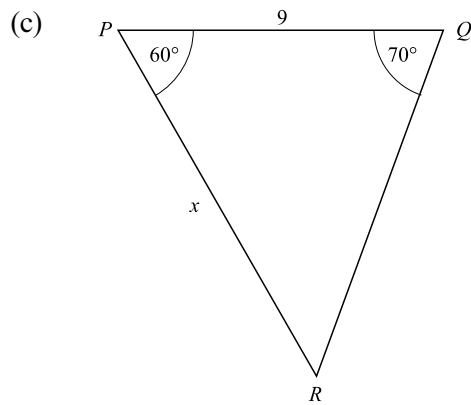
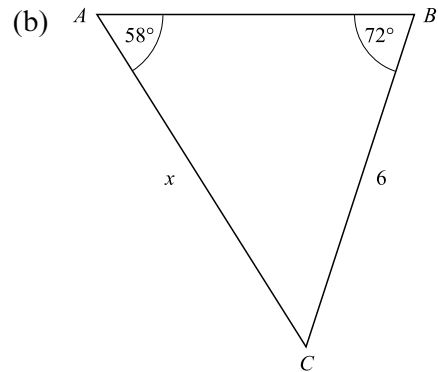
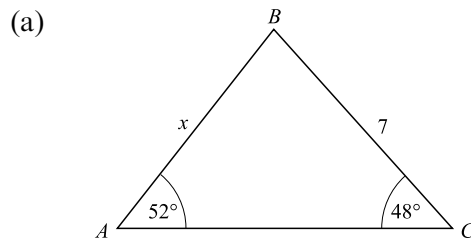




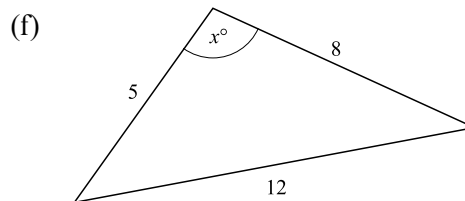
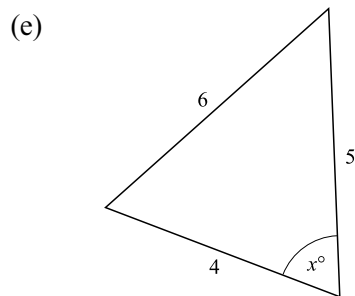
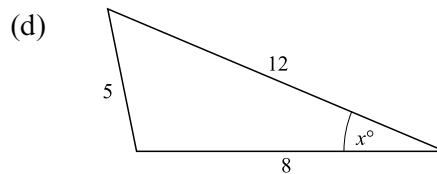
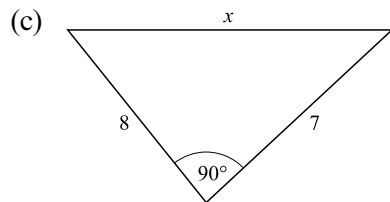
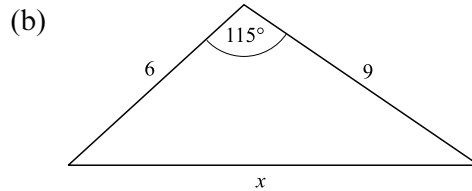
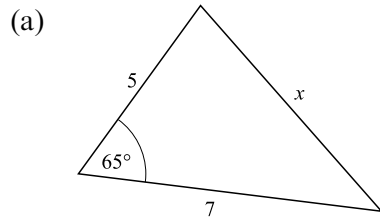
- 3** (a) Find the lengths of (i)  $BC$ , (ii)  $AB$  giving your answer in the form  $\sqrt{a}$ .  
 (b) Write down exact values for  
 (i)  $\sin 45^\circ$ , (ii)  $\cos 45^\circ$ , (iii)  $\tan 45^\circ$ .



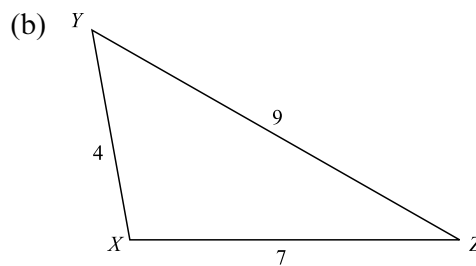
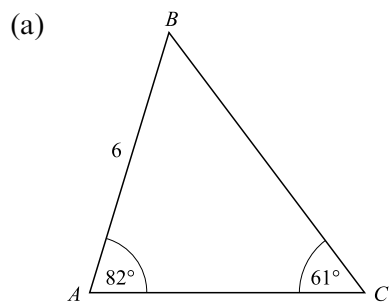
- 4** Use the sine rule to find the value of  $x$ .



5 Use the cosine rule to find the value of  $x$ .



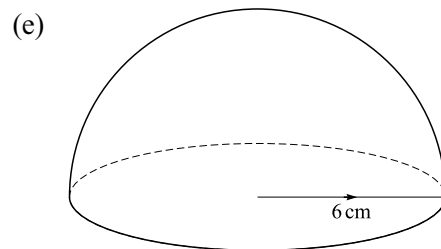
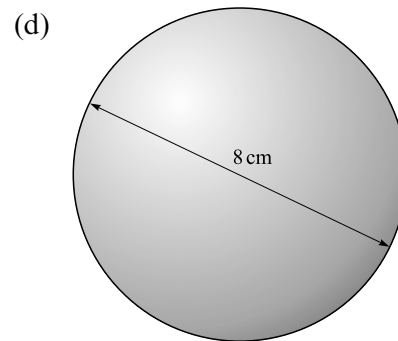
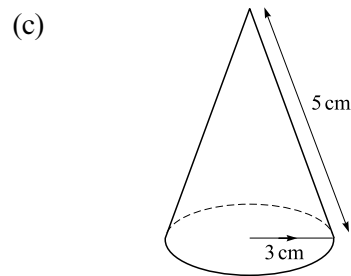
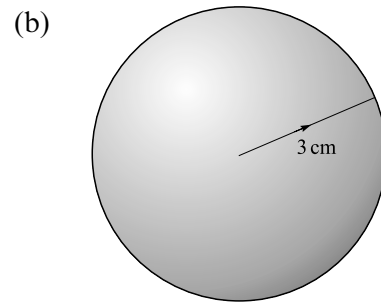
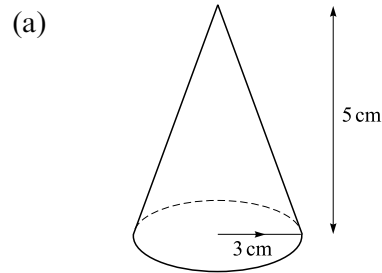
6 Use appropriate methods to find all sides and angles for:



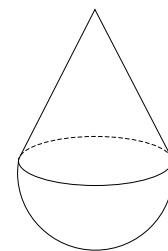
### 0.9 The cone and sphere

Volume of cone = $\frac{1}{3}\pi r^2 h$ ,	Volume of sphere = $\frac{4}{3}\pi r^3$
Surface area of cone = $\pi r \ell$ ,	Surface area of sphere = $4\pi r^2$

- 1 Find the volumes of the following solid objects, giving your answers as multiples of  $\pi$ .

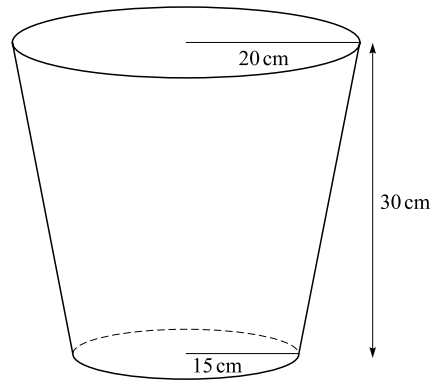


- 2 A child's toy is formed by attaching a cone to a hemisphere as shown. The radius of the hemisphere is 6 cm and the height of the toy is 14 cm. Find (a) its volume, (b) its surface area.



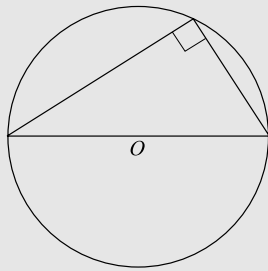
- 3 The earth may be treated as a sphere of radius 6370 km. Find (a) its surface area, (b) its volume.
- 4 Twelve balls, each of radius 3 cm, are immersed in a cylinder of water, radius 10 cm, so that they are each fully submerged. What is the rise in the water level?

- 5 A solid metal cube of side 4 cm is melted down and recast as a sphere. Show that its radius is  $\sqrt[3]{48/\pi}$ .
- 6 A gas balloon, in the shape of a sphere, is made from  $1000\text{ m}^2$  of material. Estimate the volume of gas in the balloon. What assumptions have you made?
- 7 A hollow sphere has internal diameter 10 cm and external diameter 12 cm. What is the volume of the material used to make the sphere?
- 8 A bucket is in the shape of the frustrum of a cone. The radius of the base is 15 cm and the radius of the top is 20 cm. Find the volume of the bucket, given that its height is 30 cm.

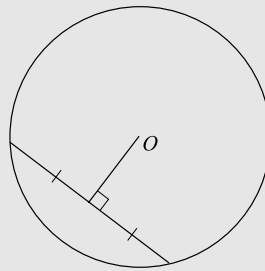


### 0.10 Properties of a circle

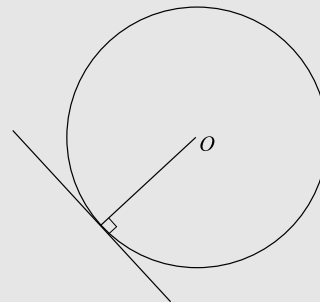
**Angle facts:**



The angle in a semi-circle is a right angle.

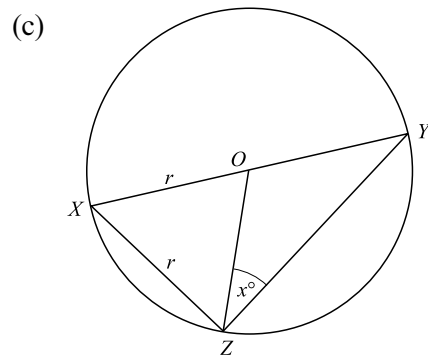
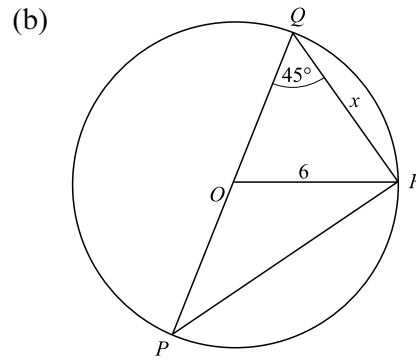
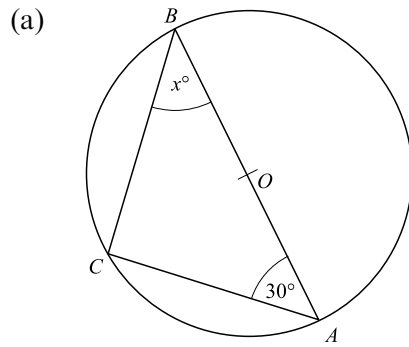


The perpendicular from the centre to a chord bisects the chord.

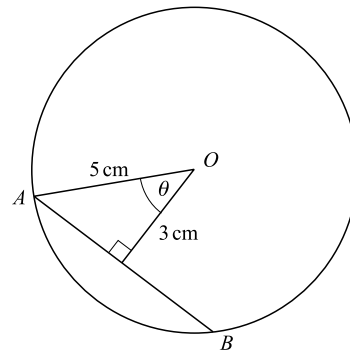


The radius is perpendicular to the tangent.

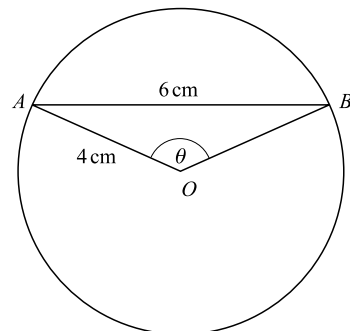
1 Find the value of  $x$  in each of the following.



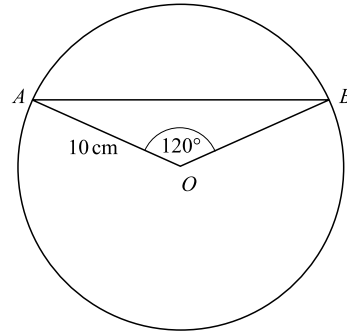
2 (a)  $AB$  is a chord of a circle, radius 5 cm, at a distance of 3 cm from the centre  $O$ . Find (i) the length  $AB$ , (ii) the angle  $\theta$ .



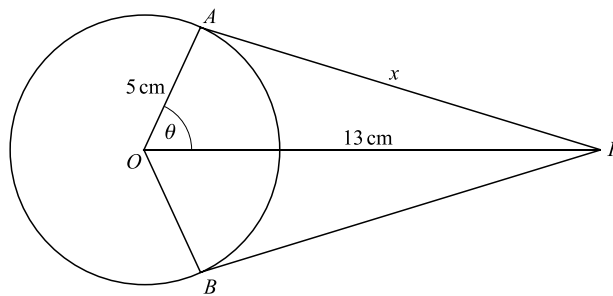
(b) Find the angle  $\theta$  subtended by the chord  $AB$  in the diagram.



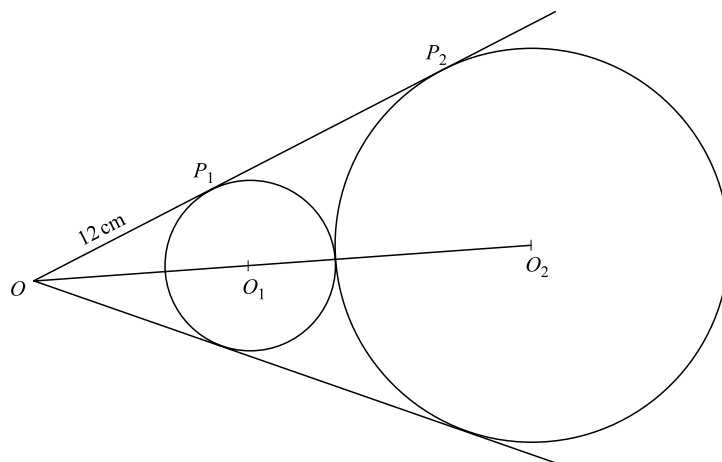
- (c) Find the area of triangle  $AOB$  and hence find the area of the minor segment cut off by  $AB$ .



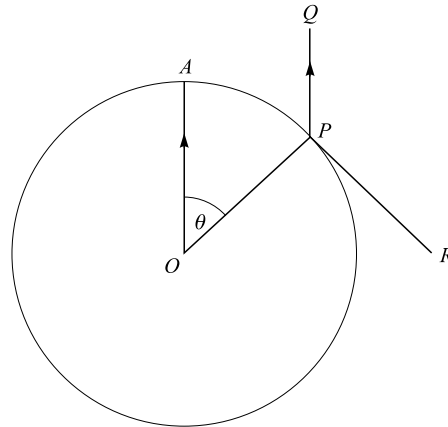
- 3 (a)  $AP$  and  $BP$  are tangents to the circle with centre  $O$  and radius 5 cm.  $OP = 13$  cm. Find (i)  $AP$ , (ii)  $\theta$ .



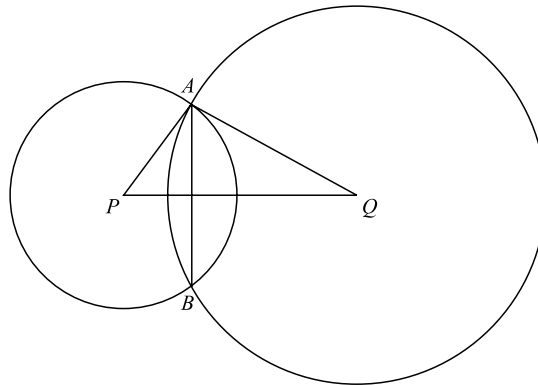
- (b)  $OP_1P_2$  is a tangent to two circles with centres  $O_1, O_2$ .  $OP_1 = 12$  cm. The radius of the circle with centre  $O_1$  is 5 cm. Find the radius of the circle with centre  $O_2$ .



- (c) In the diagram,  $OA$  is parallel to  $PQ$ . Find the angle  $QPR$  in terms of  $\theta$ .



- 4 Two circles, radii 3 cm and 5 cm, have centres  $P$ ,  $Q$  respectively,  $PQ = 7$  cm. If the circles intersect at  $A$  and  $B$ , find the length  $AB$ .



- 5 The distance from the Earth to the sun is  $1.50 \times 10^8$  km. The diameter of the sun is  $1.39 \times 10^6$  km. Find the angle subtended by the sun from a point on the Earth. What assumptions have you made?